Unit - II

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Section - A

Unit - II

Bisection method

Ex-I

Find the root of the eqⁿ. $x^3 - x^2 + 1$ by using bi-section method.

Solⁿ

Here

$$f(x) = x^3 - x^2 + 1 \quad \text{--- (1)}$$

$$x_0 = 0 \quad , \quad x_1 = -1$$

$\because$ $f(x_0)$ or $f(0) = (0)^3 - (0)^2 + 1 = 1$

$f(x_1)$ or $f(-1) = (-1)^3 - (-1)^2 + 1 = -1$

$\Rightarrow$ $f(x_0) \cdot f(x_1) < 0$

or

$f(0) \cdot f(-1) < 0$

The root of the eqⁿ (1) lies b/w $x_0 = 0$ and $x_1 = -1$ i.e. lies b/w 0 and -1

By using Bisection method

$$x_2 = \frac{x_1 + x_0}{2} = \frac{0 - 1}{2} = -0.5$$

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$f(x_2)$ or $f(-0.5) = 0.625$

So,

$f(x_1) \cdot f(x_2) < 0$

The root of the eqn (1) lies b/w -1 and -0.5

i.e. $(-1, -0.5)$

$$x_3 = \frac{x_1 + x_2}{2} = \frac{-1 + (-0.5)}{2} = \frac{-1.5}{2}$$

$$= -0.75$$

$f(x_3) = f(-0.75) = 0.015625$

Ex-II Find the root of the equation $\cos x = xe^x$ using bi-section method correct to four decimal places.

Sol^n

Let $f(x) = \cos x - xe^x = 0$ — (1)

$x_0 = 0$, $x_1 = 1$

$f(x_0) = f(0) = 1$

$f(x_1) = f(1) = -2.18$

$\Rightarrow f(x_0) \cdot f(x_1) < 0$

The root of the eq^n (1) lies b/w [0, 1]

By using bi-section method.

$x_2 = \frac{x_0 + x_1}{2} = \cancel{\frac{1 + (-2.18)}{2}} \quad \frac{0+1}{2} = 0.5$

$f(x_2) = \cos(0.5) - (0.5) \cdot e^{0.5} = 0.05$

$f(x_2) \cdot f(x_1) < 0$

$\therefore$ The root of the eq^n (1) lies b/w [1, 0.5]

$x_3 = \frac{x_1 + x_2}{2} = \frac{1+0.5}{2} = 0.75$

$f(x_3) = f(0.75) = -0.86$

$f(x_3) \cdot f(x_2) < 0$

The root of the eq^n(1) lies b/w
$f(x_2) = 0.05$ and $f(x_3) = -0.86$
i.e. lies b/w $[0.5, 0.75]$

$$x_4 = \frac{x_2 + x_3}{2} = \frac{0.5 + 0.75}{2}$$
$$= 0.625$$

$f(x_4) = f(0.625) = -0.36$

$f(x_2) \cdot f(x_4) < 0$

The root of the eq^n(1) lies b/w $[0.5, 0.625]$

$$x_5 = \frac{x_2 + x_4}{2} = \frac{0.5 + 0.625}{2}$$
$$= 0.5625$$

$f(x_5) = -0.14$
$f(x_5) \cdot f(x_2) < 0$

The root of the eq^n(1) lies b/w $[0.5, 0.5625]$

$$x_6 = \frac{0.5 + 0.5625}{2} = 0.5312$$

$f(x_6) = -0.04$
$f(x_2) \cdot f(x_6) < 0$
The root of eq^n lies b/w $[0.5, 0.5312]$

$$x_7 = \frac{0.5 + 0.5312}{2} = \boxed{0.5156}$$

Hence the desired approximation to the root is 0.5156

Newton's Raphson method

Ex-1 Find the positive root of $x^4 - x = 10$ correct to three decimal places, using Newton Raphson method

Sol^n :-

Let
$$f(x) = x^4 - x - 10 = 0$$
$$f'(x) = 4x^3 - 1$$

$f(1) = -10 = -ve$ [must both -ve and +ve]
$f(2) = 16 - 2 - 10 = 4 = +ve$

$\therefore$ A root of $f(x) = 0$ lies b/w 1 and 2

Let us take
$$x_0 = 2$$

Newton Raphson formula is

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

Putting $n = 0$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 2 - \frac{4}{4(2)^3 - 1} = 2 - \frac{4}{31}$$

$= 1.871$

$\therefore x_1 = 1.871$

Putting $n=1$, the second approximation is

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1.871 - \frac{f(1.871)}{f'(1.871)}$$

$$= 1.871 - \frac{0.3835}{25.199} = 1.856$$

Putting $n=2$, the third approx. is

$(x_{2+1}) = ~~x~~_3$ i.e. $x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$$= 1.856 - \frac{f(1.856)}{f'(1.856)}$$

$$= 1.856 - \frac{0.010}{24.574} = 1.856$$

Hence the desired root is 1.856 correct to three decimal places.

$\sin 0.6 = 0.5729.$
$\cos 0.6 = 0.82533$

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Ex-II Find by Newton's method, the real root of the eqn $3x = \cos x + 1$ correct to four decimal places

Soln :

Let $f(x) = 3x - \cos x - 1$ —(1)

$f'(x) = 3 + \sin x$

$f(0) = -2$, (-ve) $f(1) = 3 - 0.5403 - 1 = 1.4597$ (+ve)

So,
the root of the eqn (1) lies b/w 0 and 1. It is nearer to 1. Let us take $x_0 = 0.6$

Newton's Raphson formula is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Putting $n = 0$, the first approximation is

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 0.6 - \frac{f(0.6)}{f'(0.6)} = 0.6071$$

Putting $n = 1$, the 2nd approximation is

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_2)}$$

P.T.O

$$x_2 = 0.6071 - \frac{f(0.6071)}{f'(0.6071)}$$

$$= 0.6071$$

Here $x_1 = x_2$,
Hence the desired root is 0.6071 correct to four decimal places.

Note :- Newton's Raphson formula

Iterative formula to find $1/N$ = $x_{n+1} = x_n(2 - Nx_n)$

" " to find $\sqrt{N}$ = $x_{n+1} = \frac{1}{2}(x_n + N/x_n)$

$\frac{1}{\sqrt{N}}$ = $x_{n+1} = \frac{1}{2}(x_n + 1/Nx_n)$.

$\sqrt[k]{N}$ = $x_{n+1} = \frac{1}{k}\left[(k-1)x_n + N/x_n^{k-1}\right]$

Fixed Point Method :-

In numerical analysis, the fixed point iteration is the method of computing fixed points of iterated functions.

$$f(x) = 0$$

$$x = \phi(x).$$

$$x^3 + x - 1 = 0$$

$$x = 1 - x^3$$

$$x + \quad 1$$

$$x(x^2 + 1) = 1$$

$$\boxed{x = \frac{1}{1+x^2}}$$

Taking initial value $x_0 = 1$

$$x_1 = \phi(x_0) \;=\; x_1 = \phi(1) = \frac{1}{1+(1)^2} = \frac{1}{2}$$

$$= 0.5$$

$$x_2 = \phi(x_1)$$

$$x_n = \phi(x_{n-1})$$

$[a = x_0, \quad b = x_1]$

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3) Bisection method :-

A numerical method for finding a root of an equation $f(x) = 0$ is known as Bisection method.

If values a and b are found such that $f(a)$ and $f(b)$ have opposite signs and function is continuous on the interval $[a, b]$, then the root of the equation $f(x)$ lies b/w the interval $[a, b]$.

We find the middle point

$$c = \frac{a+b}{2}.$$

There are three possibilities

i) If $f(c) = 0$, then c is the root of the given eq^n.

ii) If $f(a) \cdot f(c) < 0$ i.e. $f(a)$ and $f(c)$ have ~~negative~~ opposite signs, then there exists a root b/w the $[a, b]$

iii) have same sign, not exist b/w ~~$[a, b]$~~ $[b, c]$

iv) Newton Raphson Method :-

This is also an iteration method and is used to find the root of an equation $f(x) = 0$

Let

$x = x_0$ be an approximate value of eqn in interval $[a, b]$

$f(x) = 0$.

if $x = x_1$, then

$f(x_1) = 0$

Now we get

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Similarly

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

Therefore By Newton Raphson method -

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

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